papers: Proc. Roy. Soc. (London) **A242**, 300 (1957); **A243**, 291 (1957); **A248**, 199, 222 (1958). For another interpretation one should consult PURCELL, reference 33.

50.A "Fluctuations of Photon Beams and Their Correlation." L. MANDL. Proc. Phys. Soc. (London) **A72**, 1037 (1958). A theoretical discussion of the phenomena of reference 49.

51. Resource Letter PL-1 on Polarized Light, W. A. SHURCLIFF. Am. J. Phys. **30**, 227 (1962). This resource letter should be consulted for references on polarization. In the present letter we are concerned with polarization primarily in regard to its role in the theory of measurement.

V. MASERS

The various types of masers provide interesting illustrations of the phenomena discussed in Sec. II–IV. They are also of interest in their own right. Only a few reviews, listed below, have been selected from the vast literature. Maser is an acronym for microwave amplification by stimulated emission of radiation.

***52.E "Optical Masers."** A. L. SHAWLOW. Sci. Am. **204**, No. 6, 52 (June 1961). This clear discussion (at an elementary level) may be useful in making vivid the difference between spontaneous and induced emission.

***53.E "The Maser."** J. P. GORDON. Sci. Am. **199**, No. 6, 42 (December 1958). Similar in level to the preceding reference.

54.I "Masers." J. WEBER. Revs. Mod. Phys. **31**, 681 (1959). This review can be read with profit by students conversant with elementary quantum mechanics.

55.A "Quantum Mechanical Amplifiers." W. E. LAMB, JR. In **Lectures in Theoretical Physics.** Edited by W. BRITTIN AND B. DOWNS. (Interscience Publishing Company, New York, 1960), Vol. II. This lucid review makes use of the quantized electromagnetic field.

56.I Elements of Maser Theory. A. A. VUYLSTEKE. (D. Van Nostrand Company, Inc., New York, 1960). A detailed text which explains the statistical and quantum mechanics necessary for an understanding of the subject.

VI. FILMS

Very few movies are available. However, two relevant films have been recently created as part of the PSSC program: **Photons** (19 min) and **Interference of Photons** (14 min). The latter is a sequel to **Photons.** These films are discussed by M. CORRELL, Am. J. Phys. **30**, 772 (1962). These films may be rented or bought ($90 each) from *Modern Learning Aids*, 3 East 54 Street, New York 22, N. Y. Also available from *MLA* is the **Franck-Hertz Experiment** (25 min). Further available films are listed by R. L. WEBER, Am. J. Phys. **29**, 222 (1961); **30**, 321 (1962).

Heat Waves and Ångström's Method

HOWARD C. BRYANT
Department of Physics, University of New Mexico, Albuquerque, New Mexico

(Received 20 June 1962; in final form 25 January 1963)

The study of the propagation of a fluctuating temperature along a copper rod proves to be an instructive experiment for our junior–senior laboratory. Temperatures are measured as a function of time and of distance along the rod; one end of which is periodically heated and cooled, while the other end is kept at constant temperature. The temperature distribution is that of a traveling wave, each frequency component of which damps out with a characteristic distance that decreases with increasing frequency of the component. From a determination of the velocity and characteristic damping distance of the fundamental, the student calculates the thermal diffusivity of copper.

INTRODUCTION

ABOUT one century ago, Ångström[1] proposed a method of measuring thermal diffusivity by means of alternately heating and cooling the end of a metal rod, and measuring the temperature along the rod as a function of time. A similar method[2] was proposed by R. W. King, in which he also varied the frequency of the "heat wave" in order to eliminate radiative effects. We have found a modification of Ångström's apparatus to be a very interesting experiment in our junior–

---

[1] See, for example, Thomas Preston, *Theory of Heat*, revised by J. Rogerson Cotter (MacMillan and Company, Ltd., London, 1904), p. 655; also M. N. Saha and B. N. Srivastava, *A Treatise on Heat* (The Indian Press, Ltd., Allahabad and Calcutta, 1935), 2nd ed., p. 348.

[2] R. W. King, Phys. Rev. 6, 77 (1915).

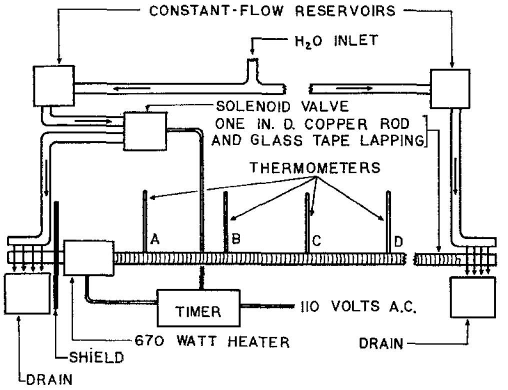

FIG. 1. A modification of Ångström's apparatus for the study of the propagation of heat waves.

senior laboratory. The apparatus is simple and inexpensive; but it demonstrates, quite well, several physical principles.

THEORY

The flow of heat is governed by the diffusion equation:

$$D\nabla^2T=\partial T/\partial t \tag{1}$$

where D, the thermal diffusivity, is the thermal conductivity divided by the product of the density and the specific heat. The Laplacian is separable in cylindrical coordinates and, if one is interested only in the dependence of T on z, the distance along the axis, and t, the time, he has

$$(\partial^2T/\partial z^2)-\alpha T=(1/D)(\partial T/\partial t), \tag{2}$$

where α depends on the heat loss per unit length through the surface, and where T is chosen so that the room temperature is zero.

If a periodic temperature "signal" of angular frequency ω is applied at $z=0$, and the rod is very long, one can show by direct substitution into Eq. (2) that the time-dependent part of the solution of the equation is

$$T=\sum_{n=1}^{\infty} C_n e^{-a_n z}\sin(n\omega t-b_n z), \tag{3}$$

where, in the case of a regular "zig-zag" input signal, for example,

$$C_n=4\Delta T_0/n^2\pi\omega \quad \text{for } n \text{ odd},$$

and

$$C_n=0 \quad \text{for } n \text{ even}, \tag{4}$$

where ΔT_0 is twice the amplitude of the signal. We also find

$$a_n=\{[(\alpha^2+n^2\omega^2/D^2)^{\frac{1}{2}}+\alpha]/2\}^{\frac{1}{2}}, \tag{5}$$

and

$$b_n=\{[(\alpha^2+n^2\omega^2/D^2)^{\frac{1}{2}}-\alpha]/2\}^{\frac{1}{2}};$$

also

$$D=\omega/2a_nb_n, \quad \text{and} \quad \alpha=a_n^2-b_n^2.$$

EXPERIMENT

Our apparatus is shown in Fig. 1. It consists of a 1-in.-diam copper rod, 1.5 m long, with thermometer wells at 15-cm intervals along the length. One end is continuously sprayed with a constant stream of water at room temperature, and the other end is fitted with a "heat wave generator" which consists of a 670-W electric heating coil wrapped around the rod, and a water nozzle. An electric timer alternately turns on the heater and the control solenoid for the water nozzle for equal intervals. No attempt is made to produce a sinusoidal temperature variation. The water used at both ends of the rod is passed through elevated constant-volume reservoirs to insure a steady flow, and to allow the tap water to reach room temperature. Most of the rod is lapped with several layers of glass tape to reduce α.

After reasonable steady-state conditions are reached, the temperatures on the four thermometers A, B, C, and D are read at half-minute

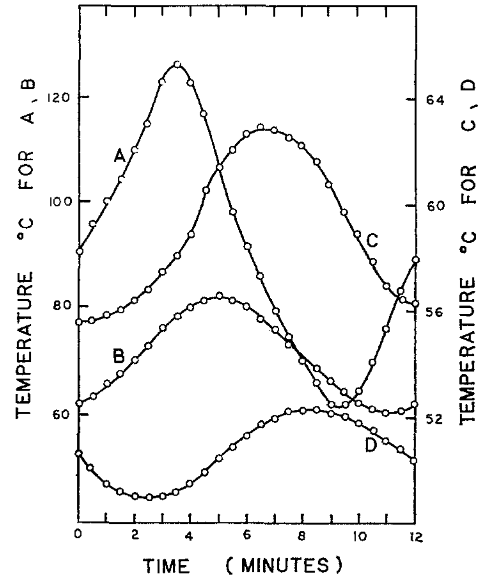

FIG. 2. Temperature vs time (arbitrary zero) at four points along the rod. Curves A and B use left ordinate; curves C and D use right ordinate.

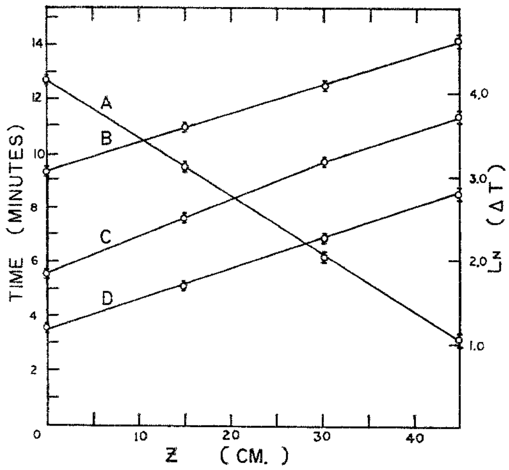

FIG. 3. A is plot of the natural logarithm of the heat wave amplitude vs distance. B,C,D are plots of the positions of the trough, median, crest vs distance.

intervals. Four or five cycles of the generator are followed until the experimenter is satisfied that the results are reproducible, and that he knows, as well as possible, the shapes and phases of the four temperature variations.

It is helpful if the period of the heat wave generator is exactly some multiple of a minute or another convenient time interval, since the measurements from successive cycles can be directly compared to check for drift; also, if the data are plotted directly, one can fill in critical points (crests, troughs, and medians[3]) more thoroughly in later cycles. The readings on thermometer A can be used as an unambiguous reference if data-taking is interrupted, as usually is the case. We used a heat-wave period of 12 min, which is a reasonable one for our dimensions.

The apparatus could be improved by using thermocouples instead of thermometers, and using ice water in the heat wave generator, although this was not done in our case.

RESULTS

The raw data from an afternoon's work are shown in Fig. 2. Note that the temperature at thermometer A differs significantly from a pure sine wave, but the curves for subsequent stations (B,C,D) approach sine waves, which demonstrates strikingly the rapid damping of the higher frequency components of the original signal.

The natural logarithm of twice the amplitude of the temperature at each station is plotted vs z (curve A, Fig. 3). A straight line fits these points, indicating that the admixture of higher frequencies does not affect appreciably the temperature amplitude even at the first station. Neglecting higher terms, then, we find from the slope of this plot:

$$a_1 = (0.074 \pm 0.001)\text{cm}^{-1}.$$

Curves B, C, and D in Fig. 3 are plots of the times (right-hand ordinate scale) of the troughs, the medians following the crests, and the crests, respectively, against z. The trough and crest plots are reasonable fits to a straight line, but the plot of the medians deviates appreciably. This latter effect is, undoubtedly, due to the higher frequency components, which show up most when the fundamental frequency component is near zero; this is the case at the median temperature for each station. The higher frequencies show up here because their velocities are different from the fundamental wave. From the slopes of these plots, we can determine the trough and crest velocity, which we take to be the velocity of the fundamental:

$$v_1 = (0.153 \pm 0.005)\text{cm/sec}.$$

Since the velocity is directly related to b_1, (i.e., $b_1 = \omega/v_1$) we can compute D[4]:

$$D = (1.04 \pm 0.04)\text{cm}^2/\text{sec}.$$

CONCLUSION

The principal result of this experiment is a clean-cut determination of the thermal diffusivity. Pedagogically, it is a good experiment because it demonstrates the results of a simple, but nontrivial, theoretical calculation. The student sees the Fourier series in action, and has an opportunity to study a damped, traveling wave at his leisure. The experiment may be done easily in one afternoon.

ACKNOWLEDGMENTS

The author is grateful to Professor Donald Skabelund for helpful suggestions, and to J. W. McLean for his help with the apparatus.

---

[3] By "median," we mean the point at which the temperature is the average of the crest and trough temperatures.

[4] This compares well with the value given by the *Handbook of Physics and Chemistry* for pure copper of 1.09 cm^2/sec.