Integration of a fin experiment into the undergraduate heat transfer laboratory

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Abstract A fin experimental apparatus was designed, developed and constructed for the undergraduate heat transfer laboratory at Purdue University at Fort Wayne. This paper presents a fin experiment that can be integrated into the undergraduate heat transfer laboratory. The objective of this experiment is to enhance the understanding of the transfer of thermal energy by undergraduate mechanical engineering students. This experiment exposes the students to several important concepts in heat transfer, such as one-dimensional, steady-state heat transfer by conduction and fin efficiency. In this type of experiment, the students get the opportunity to compare the measured temperature profiles in the fin to both analytical and numerical solutions. The experimental apparatus required to carry out this experiment is simple and inexpensive.

Keywords fins; experiment; Laboratory; heat transfer

Notation

A_c cross-sectional area
A_f surface area of fin
h convection heat transfer coefficient
k thermal conductivity
L length of fin
n nodal point
P perimeter of fin
Q_f rate of heat loss from fin
T temperature
T_o base temperature
T_∞ ambient temperature
x axial location
η fin efficiency
$\theta(x)$ excess temperature $(T(x) - T_\infty)$
θ_o $T_o - T_\infty$

Introduction

Heat transfer is a very important subject and has long been an essential part of mechanical engineering curricula all over the world. Heat transfer is encountered in a wide variety of engineering applications where heating and cooling are required. Heat transfer plays an important role in the design of many devices, such as spacecraft, radiators, heating and air conditioning systems, refrigerators and power plants.

It is important for engineers to understand the principles of thermodynamics (especially the first and second laws) and of heat transfer, and to be able to use the rate equations that govern the amount of energy being transferred via the three different modes of heat transfer (i.e., conduction, convection, and radiation). However, the majority of students perceive thermodynamics and heat transfer as difficult subjects. A portable experimental apparatus (refrigeration system) was designed, developed and constructed by Abu-Mulaweh [1] to demonstrate processes and systems which are fundamental to an understanding of the basic concepts of thermodynamics, such as the first and second laws. Similarly, the integration of the present experiment into the undergraduate heat transfer laboratory would enhance and add another dimension to the teaching and learning of the subject of heat transfer. The students would be able to apply to real-life situations the principles of conductive heat transfer, such as Fourier's law, fin efficiency, and others that they have learned in classroom lectures. This approach could make the subject of heat transfer a more pleasant experience for undergraduate mechanical engineering students.

Purdue University at Fort Wayne is a state-supported institution. This makes the purchase of new instructional laboratory apparatus a challenge, due to typical budgetary limitations. Moreover, the apparatus designed by companies specializing in education equipment may not exactly reflect the educational objectives intended by the faculty. These obstacles had forced the author to seek different ways to acquire a fin experimental laboratory apparatus for demonstrating some heat transfer principles. The author concluded that such an apparatus can be designed, developed and constructed 'in house' within a manageable budget.

Extended surfaces (fins) are used to enhance the heat transfer rate between a solid surface and adjoining fluid. The fin is a good application that involves combined conduction and convection effects. In this experiment the fin is assumed to be infinitely long (i.e., the tip of the fin is at the same temperature as the adjacent fluid) and the heat flow is one dimensional. Under steady-state conditions, the students estimate the convective heat transfer coefficient from the measured temperature profile of the fin and the rate of heat dissipation from it. The measured temperature profile of the fin is compared with both the analytical and numerical (finite-difference) solutions. The fin's efficiency is also evaluated.

Experimental apparatus and procedure

A portable fin experimental apparatus was designed and constructed. The experimental apparatus is relatively simple and inexpensive. It consists of a constant-temperature energy source (a heated aluminum plate) into which the end of a circular cross-section rod is attached, as shown in Fig. 1. An aluminum circular cross-section test rod (90 cm in length) with a diameter of 9.5 mm is employed as the fin. This 90 cm length was chosen, based on analytical calculations, to ensure that the tip of the fin is at the same temperature as the adjacent fluid. Nine copper–constantan thermocouples are inserted at equal intervals into the rod to measure the axial temperature distributions. The heated aluminum plate is made of four composite layers that are held together by screws. The upper layer is an aluminum plate

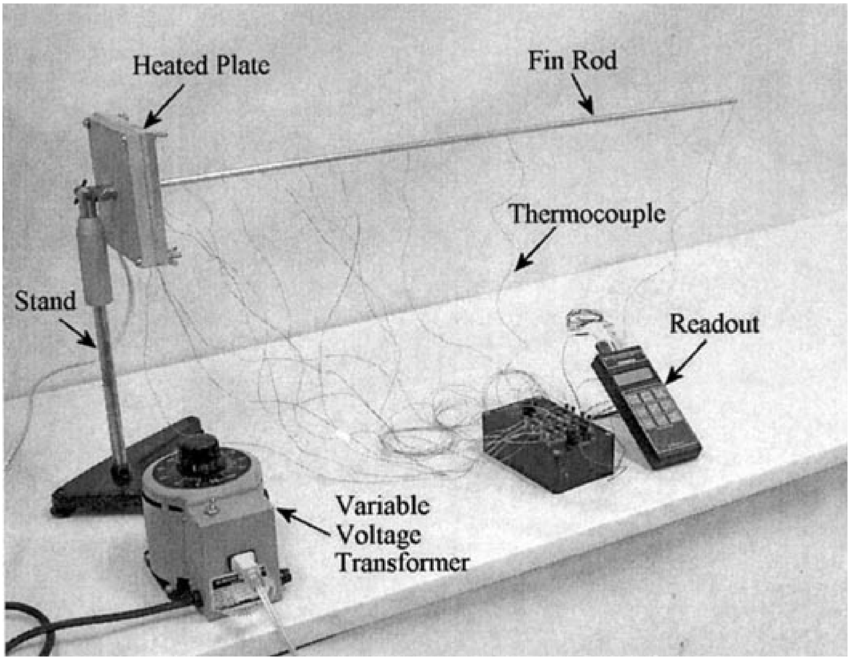

Fig. 1 *A picture of the fin apparatus.*

(15.24 cm × 15.24 cm, and 0.95 cm thick). The second layer consists of a heating pad that can be controlled for electrical energy input. The third layer is a 0.64 cm thick Transite insulating material. The bottom layer of the heated plate is aluminum plate 1.6 cm thick serving as backing and support for the heated plate structure. The aluminum plate can be heated and maintained at a constant temperature by adjusting and controlling the level of electrical energy input to the heating pad. This is accomplished utilizing a variable-voltage transformer. The temperature of the aluminum plate is measured by a thermocouple that is inserted into the plate from the back. The heated aluminum plate and the fin attachment assembly are rigidly mounted on a short stand, as shown in Fig. 1. A portable thermocouple is also used to measure the ambient temperature (i.e., air temperature). The heat is dissipated by the fin to the adjacent air by natural convection.

The experimental procedure is very simple, quick, and straightforward to carry out. First, turn on the energy source and adjust the electrical energy input to the desired heating level using the variable-voltage transformer. Second, when the system reaches steady-state conditions, measure the axial temperature distribution, $T(x)$, of the rod. In addition, measure the surrounding air temperature, T_∞, using a portable thermocouple.

Theory

Analytical solution

The analysis for fins of uniform cross-sectional area can be found in any standard heat transfer textbook (see, for example, Incropera and DeWitt [2] and Özisik [3]). In this experiment, the fin is assumed to be infinitely long (i.e., the tip of the fin is at the same temperature as the adjacent fluid) and the temperature at the base ($x = 0$) is constant, T_o. The analysis is simplified by the following assumptions: one-dimensional conduction in the x direction, steady-state conditions, constant thermal conductivity, no heat generation, constant and uniform convection heat transfer coefficient over the entire surface, and negligible radiation from the surface. Under these assumptions, the system of energy equation and boundary conditions assumes the form (refer to Fig. 2):

$$\frac{d^2T}{dx^2} - m^2(T - T_\infty) = 0 \quad (1)$$

$$T(0) = T_o \quad (2)$$

$$T(L \to \infty) = T_\infty \quad (3)$$

Where $m = \sqrt{\frac{hP}{kA_c}}$

The resulting temperature distribution and total heat transfer by the fin are given, respectively, by:

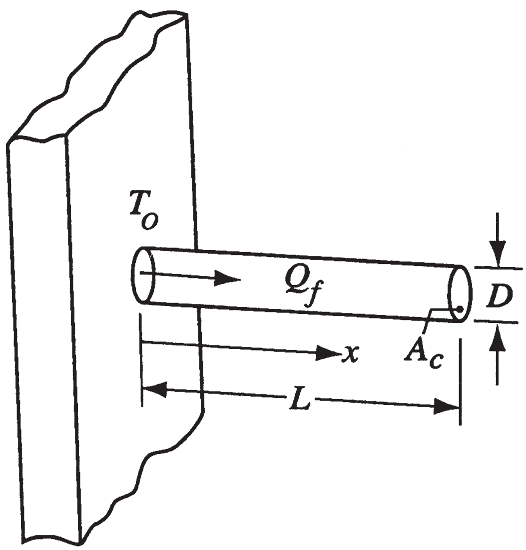

Fig. 2 *Schematic of the fin.*

$$T(x) - T_\infty = (T_o - T_\infty)\exp(-mx) \tag{4}$$

$$Q_f = \sqrt{hPkA_c}(T_o - T_\infty) \tag{5}$$

The fin efficiency, η, is defined as the ratio of the actual heat transfer through the fin to the ideal heat transfer through the fin if the entire fin surface were at fin base temperature. For the present experiment, the fin efficiency is expressed by:

$$\eta_{\text{longfin}} = \frac{1}{mL} \tag{6}$$

Finite-difference solution

The numerical method solution is employed to determine the temperature distribution in the fins. The numerical scheme that the students are asked to use is the finite-difference method. The finite-difference numerical scheme is described by Chapra and Canale [4]. In this method, the differential equation of heat conduction is approximated by a set of algebraic equations for temperature at a number of nodal points. Therefore, the first step in the analysis is the transformation of the differential equation of heat conduction in the fin into a set of algebraic equations (i.e., the finite-difference representation of the differential equation). This can be done considering an energy balance for a typical internal node of the fin rod. It should be noted that the temperatures at the boundaries are prescribed; that is $T(0) = T_o$ and $T(L) = T_\infty$. The rod is divided into N subregions, each $\Delta x = L/N$, and the node temperature is denoted by T_n, $n = 0, 1, 2, \ldots, N$, as shown in Fig. 3. The resulting general form of the finite-difference equation for the internal nodes (i.e., $n = 1, 2, \ldots, N - 1$) is:

$$\frac{kA_c}{\Delta x}(T_{n-1} - T_n) + \frac{kA_c}{\Delta x}(T_{n+1} - T_n) + hP\Delta x(T_\infty - T_n) = 0 \tag{7}$$

The simultaneous algebraic equations for temperatures at the nodal points can be solved by the Gaussian elimination method, Gauss–Seidel iteration, or by the matrix

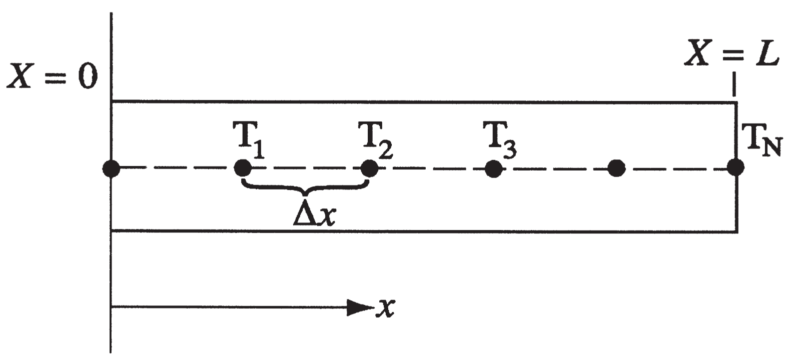

Fig. 3 *Notation for finite-difference nodes.*

inversion method. Computer programs for the solution of the simultaneous algebraic equations using these schemes are found in Özisik [3].

Estimation of the convection heat transfer coefficient

The convection heat transfer coefficient, in general, varies along the fin as well as its circumference. For convenience in the analysis, however, it is assumed that the convection heat transfer coefficient is constant and uniform over the entire surface of the fin. The value of the convection heat transfer coefficient is needed in order to determine the temperature distribution in the fin using the analytical and finite-difference solutions as given by equations 4 and 7, respectively. There are two possible methods for the estimation of the convection heat transfer coefficient in this situation. The first method utilizes equation 4 and the measured temperature distribution along the fin. This is accomplished by rearranging equation 4 and taking the natural logarithm of both sides. This yields:

$$\ln\left[\frac{T - T_\infty}{T_o - T_\infty}\right] = -mx \tag{8}$$

Use the measured temperature distribution and plot-$\ln\left[\frac{T - T_\infty}{T_o - T_\infty}\right]$ against x and then use the least squares method to find the slope of the line which is equal to m. Once the value of m is known, the convection heat transfer coefficient value can be calculated from the following relation:

$$m = \sqrt{\frac{hP}{kA_c}}$$

Also, the value of the convection heat transfer coefficient, h, can be estimated by considering an energy balance for a differential volume element $\Delta x/2$ at node $n = 0$, as shown in Fig. 4:

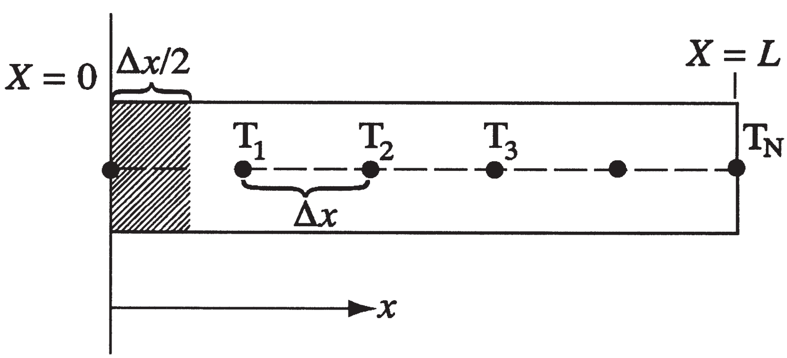

Fig. 4 *Notation for energy balance at node* n = *0.*

$$Q = \frac{kA_c}{\Delta x}(T_o - T_1) + hP\frac{\Delta x}{2}(T_o - T_\infty) \tag{9}$$

where Q is the heat entering the element from the base, which is equal to the heat lost by the fin to the adjacent fluid by convection as given by equation 5, and T_0, T_1, and T_∞ are from the measured data. Combining equations 5 and 9 yields:

$$\sqrt{hPkA_c}(T_o - T_\infty) = \frac{kA_c}{\Delta x}(T_o - T_1) + hP\frac{\Delta x}{2}(T_o - T_\infty) \tag{10}$$

Equation 10 can be solved for the convection heat transfer coefficient, h, by trial and error.

Sample results and discussion

Measurements of the axial temperature distribution along the fin were carried out for four different base temperatures (θ_o = 73.8, 90.1, 109.2, and 132.1 °C). For all of these conditions, the temperature at the tip of the fin was the same as that of the air.

The convection heat transfer coefficient value was estimated using the least squares method as shown in Fig. 5. In Fig. 5, the measured data are represented by the circles and the solid line represents the best fit using least squares. The value of

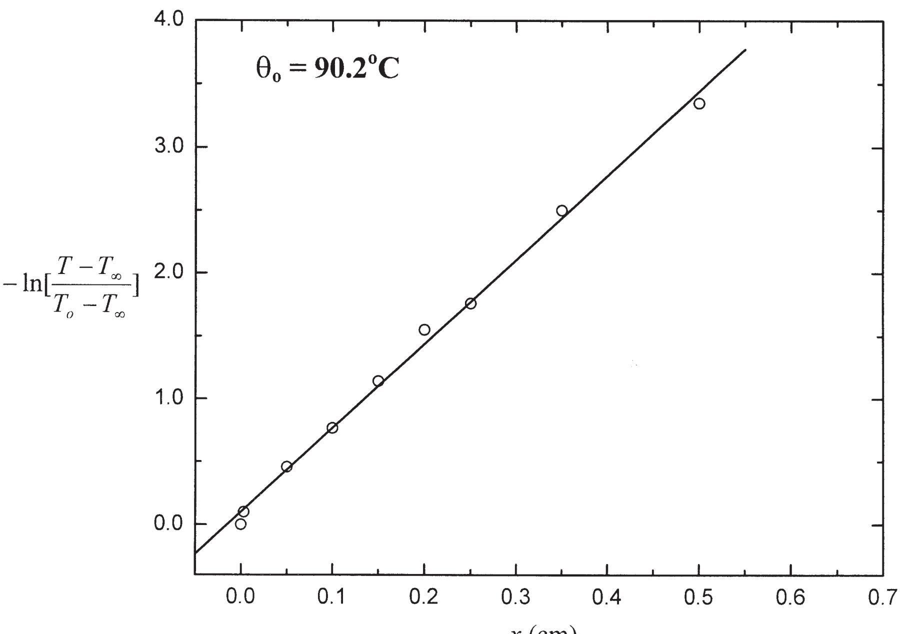

Fig. 5 *Least squares method for estimation of the convection heat transfer coefficient.*

the convection heat transfer coefficient was estimated to be h = 11.6 W/m^2.°C. And subsequently the rate of heat dissipation by the fin and its efficiency were determined using equations 5 and 6, respectively. The efficiency of the fin in the present experimental set-up was estimated at η = 16.6%. The variations in the rate of heat dissipation by the fin with the base temperature are presented in Fig. 6. Fig. 6 shows, as expected, that the rate of heat loss by the fin increases linearly as the base temperature increases. It should be noted that once the fin efficiency is known, the rate of heat dissipation by the fin can be obtained from the following relation:

$$Q_f = \eta h A_f \theta_o \tag{11}$$

The measured and the predicted dimensionless temperature distributions for the four different base temperatures (θ_o = 73.8, 90.1, 109.2, and 132.1 °C) are presented in Fig. 7. In Fig. 7, the solid lines represent the results from the analytical solution (equation 4), and the points represented by the + and circle symbols are, respectively, the predicted results from the finite-difference solution (equation 7) and the measured values. As can be seen, the measured results agree favorably with both the

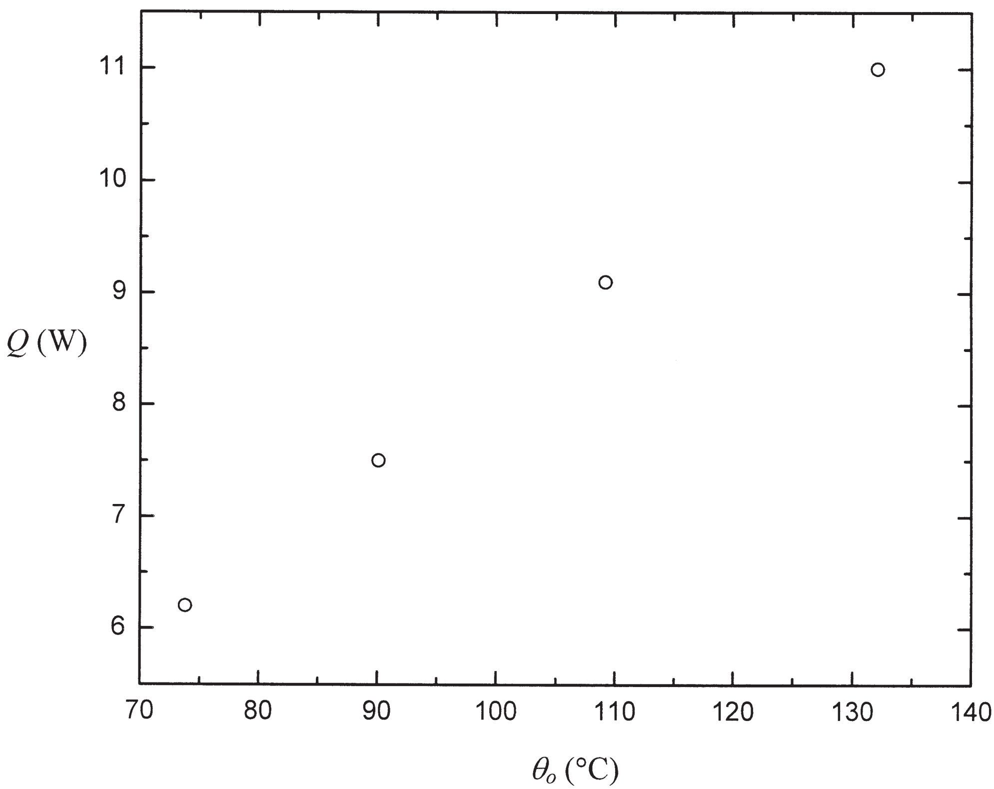

Fig. 6 *Rate of heat dissipation by the fin.*

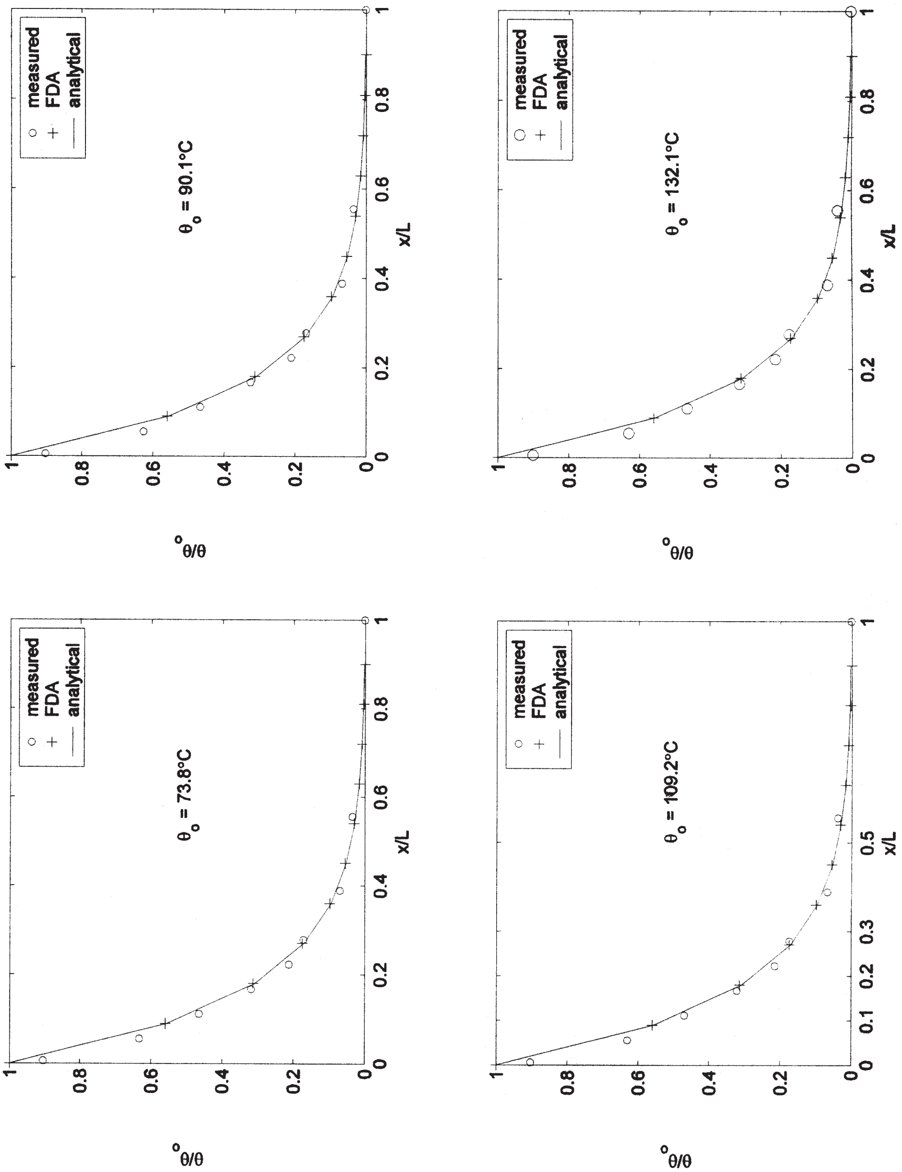

Fig. 7 *Dimensionless temperature distribution along the fin.*

analytical and the numerical solutions. The reason for carrying out the experiment at four different base temperatures is to show that the measured results are repeatable. The good agreement between the measured results and the analytical and numerical solutions and the repeatability of the experimental results indicate that the apparatus is well designed for its intended purpose of demonstrating basic heat transfer principles.

Conclusion

The experimental fin apparatus described in this paper is a valuable addition to the undergraduate heat transfer laboratory. The experimental set-up is relatively simple and the equipment relatively inexpensive and available in almost all undergraduate heat transfer laboratory. The sample results prove that the apparatus is well designed for its intended purpose of demonstrating basic heat transfer principles. The experimental apparatus is portable. This allows it to be used for laboratory experiments and classroom demonstrations. The integration of this experiment into the undergraduate heat transfer laboratory would enhance and add another dimension to the teaching and learning of the subject of heat transfer.

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