

**Colorado Virtual Campus of Physics**  
**Mechanics & Nonlinear Dynamics Cluster**  
[Nonlinear Dynamics & Chaos Lab](#)

## Instructional Experiments on Nonlinear Dynamics & Chaos (and some related theory papers)

- [overviews of nonlinear & chaotic dynamics](#)
- [prototypical nonlinear equations and their simulation](#)
- [analysis of data from chaotic systems](#)
- [control of chaos](#)
- [fractals](#)
- [solitons](#)
- [chaos in Hamiltonian/nondissipative systems & Lagrangian chaos in fluid flow](#)
- [quantum chaos](#)
- [nonlinear oscillators, vibrations & strings](#)
- [chaotic electronic circuits](#)
- [coupled systems, mode interaction & synchronization](#)
- [bouncing ball, dripping faucet, kicked rotor & other discrete interval dynamics](#)
- [nonlinear dynamics of the pendulum](#)
- [inverted pendulum](#)
- [swinging Atwood's machine](#)
- [pumping a swing](#)
- [parametric instability](#)
- [instabilities, bifurcations & catastrophes](#)
- [chemical and biological oscillators & reaction/diffusions systems](#)
- [other pattern forming systems & self-organized criticality](#)
- [miscellaneous nonlinear & chaotic systems](#)

### **-overviews of nonlinear & chaotic dynamics**

[To top?](#)

Briggs, K. (1987), "Simple experiments in chaotic dynamics," Am. J. Phys. **55** (12), 1083-9.

Hilborn, R. C. (2004), "Sea gulls, butterflies, and grasshoppers: a brief history of the butterfly effect in nonlinear dynamics," Am. J. Phys. **72** (4), 425-7.

Hilborn, R. C. and N. B. Tufillaro (1997), "Resource Letter: ND-1: nonlinear dynamics," Am. J. Phys. **65** (9), 822-34.

Laws, P. W. (2004), "A unit on oscillations, determinism and chaos for introductory physics students," Am. J. Phys. **72** (4), 446-52.

Sungar, N., J. P. Sharpe, M. J. Moelter, N. Fleishon, K. Morrison, J. McDill, and R. Schoonover (2001), "A laboratory-based nonlinear dynamics course for science and engineering students," Am. J. Phys. **69** (5), 591-7.

Gitterman, M. (2002), "Order and chaos: are they contradictory or complementary?," Eur. J. Phys. **23** (2), 119-22.

Macdonald, N. and R. R. Whitehead (1985), "Introducing students to nonlinearity," Eur. J. Phys. **6**, 143-147.

---

### **-prototypical nonlinear equations and their simulation**

[To top?](#)

Drish, W. F., Jr. and W. J. Wild (1983), "Numerical solutions of Van der Pol's equation," Am. J. Phys. **51**, 439-445.

Giambo, S., P. Pantano, and P. Tucci (1984), "An electrical model for the Korteweg-de Vries equation," Am. J. Phys. **52**, 238-243.

Olson, C. L. and M. G. Olsson (1991), "Dynamical symmetry breaking and chaos in Duffing's equation," Am. J. Phys. **59**, 907-911.

Scott, A. C. (1969), "A nonlinear Klein-Gordon equation," Am. J. Phys. **37**, 52-61.

---

### **-analysis of data from chaotic systems**

[To top?](#)

Cohen, Y., S. Katz, A. Peres, E. Santo, and R. Yitzhaki (1988), "Stroboscopic views of regular and chaotic orbits," Am. J. Phys. **56**, 1042.

de Souza-Machado, S., R. W. Rollins, D. T. Jacobs, and J. L. Hartman (1990), "Studying chaotic systems using microcomputer simulations and Lyapunov exponents," Am. J. Phys. **58**, 321-329.

Earnshaw, J. C. and D. Haughy (1993), "Lyapunov exponents for pedestrians," Am. J. Phys. **61**, 401-407.

Mitchell, T. and P. B. Siegel (1993), "A simple setup to observe attractors in phase space," Am. J. Phys. **61** (9), 855-6.

Kodba, S., M. Perc, and M. Marhl (2005), "Detecting chaos from a time series," Eur. J. Phys. **26** (1), 205-15.

Page, A., P. Candelas, and F. Belmar (2006), "On the use of local fitting techniques for the analysis of physical dynamic systems," *Eur. J. Phys.* **27** (2), 273-9.

Perc, M. (2005), "Nonlinear time series analysis of the human electrocardiogram," *Eur. J. Phys.* **26** (5), 757-68.

Perc, M. (2005), "The dynamics of human gait," *Eur. J. Phys.* **26** (3), 525-34.

## **-control of chaos**

[To top?](#)

Baker, G. L. (1995), "Control of the chaotic driven pendulum," *Am. J. Phys.* **63** (9), 832-838.

Corron, N. J., S. D. Pethel, and B. A. Hopper (2004), "A simple electronic system for demonstrating chaos control," *Am. J. Phys.* **72** (2), 272-6.

Flynn, C. and N. Wilson (1998), "A simple method for controlling chaos," *Am. J. Phys.* **66** (8), 730-5.

Gauthier, D. J. (2003), "Resource letter: CC-1: controlling chaos," *Am. J. Phys.* **71** (8), 750-9.

Starrett, J. and R. Tagg (1995), "Control of a chaotic parametrically driven pendulum," *Phys. Rev. Lett.* **74** (11), 1974-7.

## **-fractals**

[To top?](#)

Gomes, M. A. F. (1987), "Fractal geometry in crumpled paper balls," *Am. J. Phys.* **55**, 649-650.

Hurd, A. J. (1988), "Resource letter FR-1: Fractals," *Am. J. Phys.* **56**, 969-975.

Lima, F. F., V. M. Oliveira, and Gomes. M. A. F. (1993), "A Galilean experiment to measure a fractal dimension," *Am. J. Phys.* **61**, 421-422.

Tufillaro, N. B. (2001), "Generating a fractal using a capacitor," *Am. J. Phys.* **69** (6), 721-2.

Uozumi, J., K. -E. Peiponen, M. Savolainen, R. Silvennoinen, and T. Asakura (1994), "Demonstration of diffraction by fractals," *Am. J. Phys.* **62** (3), 283-5.

Bucher, J. P. (1991), "Magnetic marbles as a model for ferromagnetic particle aggregation: fractal dimensions," *Eur. J. Phys.* **12**, 142-145.

Rage, T., V. Frette, G. Wagner, T. Walmann, K. Christensen, and Tao Sun (1996), "Construction of a DLA cluster model," *Eur. J. Phys.* **17** (3), 110-15.

---

## **-solitons**

[To top?](#)

Bettini, A., T. A. Minelli, and D. Pascoli (1983), "Solitons in undergraduate laboratory," *Am. J. Phys.* **51**, 977-984.

Degasperis, A. (1998), "Resource Letter Sol-1: Solitons," *Am. J. Phys.* **66** (6), 486-97.

Giambo, S., P. Pantano, and P. Tucci (1984), "An electrical model for the Korteweg-de Vries equation," *Am. J. Phys.* **52**, 238-243.

Laroche, C., T. Dauxois, and M. Peyrard (2000), "Discreteness effects on soliton dynamics: A simple experiment," *Am. J. Phys.* **68** (6), 552-5.

Laws, P. W. (2004), "A unit on oscillations, determinism and chaos for introductory physics students," *Am. J. Phys.* **72** (4), 446-52.

Olsen, M., H. Smith, and A. C. Scott (1984), "Solitons in a wave tank," *Am. J. Phys.* **52**, 826-830.

Whitehead, J. A. (1987), "A laboratory demonstration of solitons using a vertical watery conduit in syrup," *Am. J. Phys.* **55**, 998-1003.

Winkler, E. and J. Wu (1990), "An experiment to study localized excitations - nonpropagating hydrodynamics solitons," *Am. J. Phys.* **58**, 1100-1104.

Kuusela, T., J. Kietarinta, K. Kokko, and R. Laiho (1987), "Soliton experiments in a nonlinear electrical transmission line," *Eur. J. Phys.* **8**, 27-33.

Kuusela, T., J. Kietarinta, K. Kokko, and R. Laiho (1987), "Soliton experiments in a nonlinear electrical transmission line," *Eur. J. Phys.* **8**, 27-33.

---

## **-chaos in Hamiltonian/nondissipative systems & Lagrangian chaos in fluid flow**

[To top?](#)

Alvarez, L. W., R. Smits, and G. Senecal (1975), "Mechanical analog of the synchrotron, illustrating phase stability and two-dimensional focusing," *Am. J. Phys.* **43**, 293-296.

Bercovich, C., U. Smilansky, and G. P. Farmelo (1991), "Demonstration of classical chaotic scattering," *Eur. J. Phys.* **12**, 122-128.

Berry, M. V. (1981), "Regularity and chaos in classical mechanics, illustrated by three deformations of a circular 'billiard'," *Eur. J. Phys.* **2**, 91-102.

Villermaux, E. and J. P. Hulin (1990), "Chaos Lagrangian et melange de fluides visqueux," *Eur. J. Phys.* **11**, 179-183.

## **-quantum chaos**

Arcos, E., G. Baez, P. A. Cuatlayol, M. L. H. Prian, R. A. Mendez-Sanchez, and H. Hernandez-Saldana (1998), "Vibrating soap films: An analog for quantum chaos on billiards," *Am. J. Phys.* **66** (7), 601-607.

[To top?](#)

## **-nonlinear oscillators, vibrations & strings**

[To top?](#)

Berger, J. E. and G. Nunes (1997), "A mechanical duffing oscillator for the undergraduate laboratory," *Am. J. Phys.* **65** (9), 841-846.

Berthet, R., A. Petrosyan, and B. Roman (2002), "An analog experiment of the parametric instability," *Am. J. Phys.* **70** (7), 744-749.

Case, W. B. (1994), "Time-delay oscillator and instability: a demonstration," *Am. J. Phys.* **62** (3), 227-30.

DeYoung, P. A., D. LaPointe, J. Levy, and W. Lorenz (1996), "Nonlinear coupled oscillators and Fourier transforms: an advanced undergraduate laboratory," *Am. J. Phys.* **64** (7), 898-902.

- Dorner, R., L. Kowalski, and M. Stein (1996), "A nonlinear mechanical oscillator for physics laboratories," *Am. J. Phys.* **64** (5), 575-80.
- Elliott, J. A. (1980), "Intrinsic nonlinear effects in vibrating strings," *Am. J. Phys.* **48**, 478-480.
- Elliott, J. A. (1982), "Nonlinear resonance in vibrating strings," *Am. J. Phys.* **50**, 1148-1150.
- Flerackers, E. L. M., H. J. Janssen, and L. Beerden (1985), "Piecewise linear anharmonic LRC circuit for demonstrating "soft" and "hard" spring nonlinear resonant behavior," *Am. J. Phys.* **53**, 575-577.
- Fulcher, L. P., R. C. Scherer, A. Melnykov, V. Gateva, and M. E. Limes (2006), "Negative Coulomb damping, limit cycles, and self-oscillation of the vocal folds," *Am. J. Phys.* **74** (5), 386-93.
- Hanggi, p. and P. Riseborough (1983), "Dynamics of nonlinear dissipative oscillators," *Am. J. Phys.* **51**, 347-352.
- Heard, T. C. and N. D. Mewby, Jr. (1977), "Behavior of a soft spring," *Am. J. Phys.* **45**, 1102-1106.
- Janssen, H. J., R. Serneels, L. Beerden, and E. L. M. Flerackers (1983), "Experimental demonstration of the resonance effect of an anharmonic oscillator," *Am. J. Phys.* **51**, 655-658.
- Mohazzabi, P. (2004), "Theory and examples of intrinsically nonlinear oscillators," *Am. J. Phys.* **72** (4), 492-8.
- Pecori, B., G. Torzo, and A. Sconza (1999), "Harmonic and anharmonic oscillations investigated by using a microcomputer-based Atwood's machine," *Am. J. Phys.* **67** (3), 228-35.
- Prosperetti, A. (1976), "Subharmonics and ultraharmonics in the forced oscillations of weakly nonlinear systems," *Am. J. Phys.* **44**, 548-554.
- Skeldon, K. D., V. J. Nadeau, and C. Adams (1998), "The resonant excitation of a wineglass using positive feedback with optical sensing," *Am. J. Phys.* **66** (10), 851-60.
- Smith, H. J. T. and K. A. Woolner (1984), "Inexpensive demonstration of an anharmonic oscillator," *Am. J. Phys.* **52**, 800-801.
- Thomsen, J. S. (1988), "A benevolent nonlinear system: the dynamically shifted oscillator," *Am. J. Phys.* **56**, 123-128.
- Tufillaro, N. B. (1989), "Nonlinear and chaotic string vibrations," *Am. J. Phys.* **57**, 408-414.
- Weltner, K., A. S. C. Esperidiao, R. F. S. Andrade, and G. P. Guedes (1994), "Demonstrating different forms of the bent tuning curve with a mechanical oscillator," *Am.*

J. Phys. **62** (1), 56-9.

Zilio, S. C. (1982), "Measurement and analysis of large-angle pendulum motion," Am. J. Phys. **50**, 450-452.

Dixon, M. (1985), "Amplitude jumps of a nonlinear oscillator," Eur. J. Phys. **6**, 72-79.

Grosu, I. and D. Ursu (1986), "Linear and nonlinear oscillations: a experiment for students," Eur. J. Phys. **7**, 91-94.

Whineray, S. (1991), "A cube-law air track oscillator," Eur. J. Phys. **12**, 90-95.

Whineray, S., C. Rofo, and A. Ardekani (1992), "The resonant response of a cube-law air track oscillator," Eur. J. Phys. **13**, 201-209.

Whineray, S., C. Rofo, and A. Ardekani (1992), "The resonant response of a cube-law air track oscillator," Eur. J. Phys. **13** (5), 2.

Lancaster, G. (1983), "Measurements of some properties of non-Hookean springs," Phys. Educ. **18**, 217-220.

---

## **-chaotic electronic circuits**

[To top?](#)

Berthet, R., A. Petrosyan, and B. Roman (2002), "An analog experiment of the parametric instability," Am. J. Phys. **70** (7), 744-749.

Carroll, T. L. (1995), "A simple circuit for demonstrating regular and synchronized chaos," Am. J. Phys. **63** (4), 377-9.

Clark, B. K., R. F. Martin, Jr., R. J. Moore, and K. E. Jesse (1995), "Fractal dimension of the strange attractor of the bouncing ball circuit," Am. J. Phys. **63** (2), 157-63.

DeSerio, R. (2004), "Synchronous analog I/O for acquisition of chaotic data in periodically driven systems," Am. J. Phys. **72** (4), 553-8.

Flerackers, E. L. M., H. J. Janssen, and L. Beerden (1985), "Piecewise linear anharmonic LRC circuit for demonstrating "soft" and "hard" spring nonlinear resonant behavior," Am. J. Phys. **53**, 575-577.

Hellen, E. H. (2004), "Real-time finite difference bifurcation diagrams from analog electronic circuits," Am. J. Phys. **72** (4), 499-502.

- Jones, B. K. and G. Trefan (2001), "The Duffing oscillator: A precise electronic analog chaos demonstrator for the undergraduate laboratory," *Am. J. Phys.* **69** (4), 464-9.
- Kiers, K., D. Schmidt, and J. C. Sprott (2004), "Precision measurements of a simple chaotic circuit," *Am. J. Phys.* **72** (4), 503-9.
- Lanzara, E., R. N. Mantegna, B. Spagnolo, and R. Zangara (1997), "Experimental study of a nonlinear system in the presence of noise: the stochastic resonance," *Am. J. Phys.* **65** (4), 341-9.
- levinsen, M. T. (1993), "The chaotic oscilloscope," *Am. J. Phys.* **61**, 155-165.
- Lewis, E. A. S. (1976), "Negative resistor to provide self-oscillation in RLC circuits," *Am. J. Phys.* **44**, 1217-1219.
- Mishina, T., T. Kohmoto, and T. Hashi (1985), "Simple electronic circuit for the demonstration of chaotic phenomena," *Am. J. Phys.* **53**, 332-334.
- Sprott, J. C. (2000), "Simple chaotic systems and circuits," *Am. J. Phys.* **68** (8), 758-63.
- Thomsen, J. S. (1988), "A benevolent nonlinear system: the dynamically shifted oscillator," *Am. J. Phys.* **56**, 123-128.
- Weldon, T. P. (1990), "An inductorless double scroll chaotic circuit," *Am. J. Phys.* **58**, 936-941.
- Wiener, R. J., K. E. Callan, S. C. Hall, and T. Olsen (2006), "Proportional feedback control of chaos in a simple electronic oscillator," *Am. J. Phys.* **74** (3), 200-6.
- Zimmerman, R. L., S. Celaschi, and L. G. Neto (1992), "The electronic bouncing ball," *Am. J. Phys.* **60**, 370-375.
- Clauss, D. A., R. M. Ralich, and R. D. Ramsier (2001), "Hysteresis in a light bulb: connecting electricity and thermodynamics with simple experiments and simulations," *Eur. J. Phys.* **22** (4), 385-94.
- Janssen, H. J., L. Beerden, and E. L. M. Flerackers (1984), "An experimental look at the resonant behavior of a nonlinear LC circuit," *Eur. J. Phys.* **5**, 94-100.
- Kodba, S., M. Perc, and M. Marhl (2005), "Detecting chaos from a time series," *Eur. J. Phys.* **26** (1), 205-15.
- Sobrinho, T., M. Alonso, V. Remez-Munusuri, and V. Perez-Villar (1993), "Reaction-diffusion process in a one-dimensional array of active non-linear circuits," *Eur. J. Phys.* **14**, 74-79.
- Tamasevicius, A., G. Mykolaitis, V. Pyragas, and K. Pyragas (2005), "A simple chaotic oscillator for educational purposes," *Eur. J. Phys.* **26** (1), 61-3.



---

## **-coupled systems, mode interaction & synchronization**

[To top?](#)

Carroll, T. L. (1995), "A simple circuit for demonstrating regular and synchronized chaos," Am. J. Phys. **63** (4), 377-9.

Denardo, B., J. Earwood, and V. Sazonova (1999), "Parametric instability of two coupled nonlinear oscillators," Am. J. Phys. **67** (3), 187-95.

Heath, T. and K. Wiesenfeld (1998), "Mutual entrainment of two nonlinear oscillators," Am. J. Phys. **66** (10), 860-6.

Jensen, R. V. (2002), "Synchronization of driven nonlinear oscillators," Am. J. Phys. **70** (6), 607-19.

Pantaleone, J. (2002), "Synchronization of metronomes," Am. J. Phys. **70** (10), 992-1000.

Shew, W. L., H. A. Coy, and J. F. Lindner (1999), "Taming chaos with disorder in a pendulum array," Am. J. Phys. **67** (8), 703-8.

van der Weele, J. P. and E. J. Banning (2001), "Mode interaction in horses, tea, and other nonlinear oscillators: The universal role of symmetry," Am. J. Phys. **69** (9), 953-65.

Charru, F. (1997), "A simple mechanical system mimicking phase transitions in a one-dimensional medium," Eur. J. Phys. **18** (6), 417-24.

---

## **-bouncing ball, dripping faucet, kicked rotor & other discrete interval dynamics**

[To top?](#)

Clark, B. K., R. F. Martin, Jr., R. J. Moore, and K. E. Jesse (1995), "Fractal dimension of the strange attractor of the bouncing ball circuit," Am. J. Phys. **63** (2), 157-63.

Dreyer, K. and F. R. Hickey (1991), "The route to chaos in a dripping water faucet," Am. J. Phys. **59**, 619-627.

Mello, T. M. and N. B. Tufillaro (1987), "Strange attractors of a bouncing ball," Am. J. Phys. **55**, 316-320.

Tufillaro, N. B. and A. M. Albano (1986), "Chaotic dynamics of a bouncing ball," Am. J. Phys. **54**, 939-44.

Zimmerman, R. L. and S. Celaschi (1988), "Comment on "Chaotic dynamics of a bouncing ball" [Am. J. Phys. 54, 939 (1986)]," Am. J. Phys. **56**, 1147-1148.

Zimmerman, R. L., S. Celaschi, and L. G. Neto (1992), "The electronic bouncing ball," Am. J. Phys. **60**, 370-375.

Núñez Yepez, H. N., A. L. Salas Brito, C. A. Vargas, and L. A. Vicente (1989), "Chaos in a dripping faucet," Eur. J. Phys. **10**, 99-105.

Schmidt, T. and M. Marhl (1997), "A simple mathematical model of a dripping tap," Eur. J. Phys. **18** (5), 377-83.

---

## **-nonlinear dynamics of the pendulum**

[To top?](#)

Baker, G. L. (1995), "Control of the chaotic driven pendulum," Am. J. Phys. **63** (9), 832-838.

Baker, G. L. (2006), "Probability, pendulums, and pedagogy," Am. J. Phys. **74** (6), 482-489.

Berdahl, J. P. a. L., K. V. (2001), "Magnetically driven chaotic pendulum," Am. J. Phys. **69** (9), 1016-1019.

Blackburn, J. A. and G. L. Baker (1998), "A comparison of commercial chaotic pendulums," Am. J. Phys. **66** (9), 821-830.

Coullet, P., J. M. Gilli, M. Monticelli, and N. Vandenberghe (2005), "A damped pendulum forced with a constant torque," Am. J. Phys. **73** (12), 1122-8.

Cross, R. (2005), "A double pendulum swing experiment: in search of a better bat," Am. J. Phys. **73** (4), 330-9.

Cuerno, R., A. F. Rañada, and J. J. Ruiz-Lorenzo (1992), "Deterministic chaos in the elastic pendulum: a simple laboratory for nonlinear dynamics," Am. J. Phys. **60**, 73-79.

Curzon, F. L., A. L. H. Loke, M. E. Lefrancois, and K. E. Novik (1995), "Parametric instability of a pendulum," Am. J. Phys. **63** (2), 132-6.

Duchesne, B., C. W. Fischer, C. G. Gray, and K. R. Jeffrey (1991), "Chaos in the motion of an inverted pendulum: an undergraduate laboratory experiment," Am. J. Phys. **59**, 987-992.

Frankl, D. R. (1994), "Comment on "Chaos in a computer-animated pendulum," by R.L. Kautz [Am. J. Phys. 61, 407-415 (1993)]," Am. J. Phys. **62** (9), 854 .

Grandy, W. T., Jr. and M. Schock (1997), "Simulations of nonlinear pivot-driven pendula,"

Am. J. Phys. **65** (5), 376-81.

Grosu, I. and D. Ursu (1982), "Simple apparatus for obtaining parametric resonance," Am. J. Phys. **50**, 561.

Hall, D. E. and M. J. Shea (1977), "Large-amplitude pendulum experiment: another approach," Am. J. Phys. **45**, 355-357.

Kautz, R. L. (1993), "Chaos in a computer animated pendulum," Am. J. Phys. **61** (5), 407-415.

Levien, R. B. and S. M. Tan (1993), "Double pendulum: an experiment in chaos," Am. J. Phys. **61** (11), 1038-44.

Marega, E., Jr., L. Ioriatti, and S. C. Zilio (1991), "Harmonic generation and chaos in an electromechanical pendulum," Am. J. Phys. **59**, 858-859.

Peters, R. D. (1995), "Chaotic pendulum based on torsion and gravity in opposition," Am. J. Phys. **63** (12), 1128-36.

Ruby, L. (1994), "Comment on "Chaos in a computer-animated pendulum" by R.L. Kautz [Am. J. Phys. 61, 407-415 (1993)]," Am. J. Phys. **62** (5), 472 .

Schery, S. D. (1976), "Design of an inexpensive pendulum for study of large-angle motion," Am. J. Phys. **44**, 666-670.

Shinbrot, T., C. Grebogi, J. Wisdom, and J. A. Yorke (1992), "Chaos in a double pendulum," Am. J. Phys. **60**, 491-499.

Siahmakoun, A., V. A. French, and J. Patterson (1997), "Nonlinear dynamics of a sinusoidally driven pendulum in a repulsive magnetic field," Am. J. Phys. **65** (5), 393-400.

Simon, R. and R. P. Riesz (1979), "Large amplitude simple pendulum: a Fourier analysis," Am. J. Phys. **47**, 898-899.

Simon, R. and R. P. Riesz (1980), "Erratum: "Large amplitude simple pendulum: a Fourier analysis" [Am. J. Phys. 47, 898-899 (1979)]," Am. J. Phys. **48**, 582.

VanDalen, G. J. (2004), "The driven pendulum at arbitrary drive angles," Am. J. Phys. **72** (4), 484-91.

Yorke, E. D. (1978), "Square-wave model for a pendulum with an oscillating suspension," Am. J. Phys. **46**, 285-288.

Aggarwal, N., N. Verma, and P. Arun (2005), "Simple pendulum revisited," Eur. J. Phys. **26** (3), 517-23.

Belendez, A., A. Hernandez, A. Marquez, T. Belendez, and C. Neipp (2006), "Analytical approximations for the period of a nonlinear pendulum," Eur. J. Phys. **27** (3), 539-51.

Denny, M. (2002), "The pendulum clock: a venerable dynamical system," *Eur. J. Phys.* **23** (4), 449-58.

Irons, F. E. (1990), "Concerning the non-linear behavior of the forced spherical pendulum including the dowsing pendulum," *Eur. J. Phys.* **11**, 107-115.

Lewowski, T. and K. Wozniak (2002), "The period of a pendulum at large amplitudes: a laboratory experiment," *Eur. J. Phys.* **23** (5), 461-4.

Milotti, E. (2001), "Nonlinear behaviour in a torsion pendulum," *Eur. J. Phys.* **22** (3), 239-48.

Parwani, R. R. (2004), "An approximate expression for the large angle period of a simple pendulum," *Eur. J. Phys.* **25** (1), 37-9.

Rousseaux, G., P. Coullet, and J. -M. Gilli (2005), "Amplitude equations for mechanical analogues of Faraday and nonlinear optical rotations," *Eur. J. Phys.* **26** (6), 1065-78.

Tritton, D. J. (1986), "Ordered and chaotic motion of a forced spherical pendulum," *Eur. J. Phys.* **7**, 162-169.

## **-inverted pendulum**

[To top?](#)

Alessi, N., C. W. Fischer, and C. G. Gray (1992), "Measurement of amplitude jumps and hysteresis in a driven inverted pendulum," *Am. J. Phys.* **60**, 755-756.

Blackburn, J. A., H. J. T. Smith, and N. Gronbech-Jensen (1992), "Stability and Hopf bifurcations in an inverted pendulum," *Am. J. Phys.* **60**, 903-908.

Blitzer, L. (1965), "Inverted pendulum," *Am. J. Phys.* **33**, 1076-1078.

Duchesne, B., C. W. Fischer, C. G. Gray, and K. R. Jeffrey (1991), "Chaos in the motion of an inverted pendulum: an undergraduate laboratory experiment," *Am. J. Phys.* **59**, 987-992.

Fenn, J. G., D. A. Bayne, and B. D. Sinclair (1998), "Experimental investigation of the "effective potential" of an inverted pendulum," *Am. J. Phys.* **66** (11), 981-4.

Friedman, M. H., J. E. Campana, L. Kelner, and E. H. Seeliger (1982), "The inverted pendulum: a mechanical analog of the quadrupole mass filter," *Am. J. Phys.* **50**, 924-931.

Jones, H. W. (1969), "A quick demonstration of the inverted pendulum," *Am. J. Phys.* **37**, 941.

Joshi, S. S. (1966), "inverted pendulum with damping," *Am. J. Phys.* **34**, 533.

Kalmus, H. P. (1970), "The inverted pendulum," *Am. J. Phys.* **38**, 874-878.

- King, R. E. (1965), "The inverted pendulum," Am. J. Phys. **33**, 855-856.
- Michaelis, M. M. (1985), "Stroboscopic study of the inverted pendulum," Am. J. Phys. **53**, 1079-1083.
- Moloney, M. J. (1996), "Inverted pendulum motion and the principle of equivalence," Am. J. Phys. **64** (1).
- Murgatroyd, P. N. (1994), "The magnetic analogue of the inverted pendulum," Am. J. Phys. **62** (3), 281-2.
- Phelps, F. M., III and J. H. Hunter, Jr. (1965), "An analytical solution of the inverted pendulum," Am. J. Phys. **33**, 285-295.
- Phelps, F. M., III and J. H. Hunter, Jr. (1966), "Reply to Joshi's comments [Am. J. Phys. 34, 533 (1966)] on a damping term in the equations of motion of the inverted pendulum," Am. J. Phys. **34**, 533.
- Smith, H. J. T. and J. A. Blackburn (1992), "Experimental study of an inverted pendulum," Am. J. Phys. **60**, 909-911.
- Mata, G. J. and E. Pestana (2004), "Effective Hamiltonian and dynamic stability of the inverted pendulum," Eur. J. Phys. **25** (6), 717-21.
- Pippard, A. B. (1987), "The inverted pendulum," Eur. J. Phys. **8**, 203-206.

### **-swinging Atwood's machine**

[To top?](#)

- Griffiths, D. J. and T. A. Abbott (1992), "Comment on 'A surprising mechanics demonstration', by A.R. Marlow [Am. J. Phys. 59, 951-952 (1991)]," Am. J. Phys. **60** (10), 951-3.
- Nunes, A., J. Casasayas, and N. Tufillaro (1995), "Periodic orbits of the integrable swinging Atwood's machine," Am. J. Phys. **63** (2), 121-6.
- Tufillaro, N. (1986), "Integrable motion of a swinging Atwood's machine," Am. J. Phys. **54** (2), 142-3.
- Tufillaro, N. B. (1994), "Teardrop and heart orbits of a swinging Atwood's machine," Am. J. Phys. **62** (3), 2.
- Tufillaro, N. B. (1994), "Teardrop and heart orbits of a swinging Atwood's machine," Am.

J. Phys. **62** (3), 231-3.

Tufillaro, N. B., T. A. Abbott, and D. J. Griffiths (1984), "Swinging Atwood's machine," Am. J. Phys. **52** (10), 895-903.

Tufillaro, N. B., T. A. Abbott, and D. J. Griffiths (1984), "Swinging Atwood's machine," Am. J. Phys. **52**, 895-903.

Tufillaro, N., A. Nunes, and J. Casasayas (1988), "Unbounded orbits of a swinging Atwood's machine," Am. J. Phys. **56** (12), 1117-20.

Tufillaro, N., A. Nunes, and J. Casasayas (1988), "Unbounded orbits of a swinging Atwood's machine," Am. J. Phys. **56**, 1117-1120.

Casasayas, J., N. Tufillaro, and A. Nunes (1989), "Infinity manifold of a swinging Atwood's machine," Eur. J. Phys. **10**, 173-177.

Casasayas, J., N. Tufillaro, and A. Nunes (1989), "Infinity manifold of a swinging Atwood's machine," Eur. J. Phys. (UK) **10** (3), 173-7.

Bruhn, B. (1987), "Chaos and order in weakly coupled systems of nonlinear oscillators," Phys. Scr. (Sweden) **35** (10), 7-12.

Casasayas, J., A. Nunes, and N. Tufillaro (1990), "Swinging Atwood's Machine: integrability and dynamics," J. Phys. (France) **51** (16), 1693-702.

Ouazzani-T. H. , A. and M. Ouazzani-Jamil (1995), "Bifurcations of Liouville tori of an integrable case of swinging Atwood's machine," Nuovo Cimento B (Italy) **110B** (9), 1111-21.

Tufillaro, N. B. (1985), "Collision orbits of a swinging Atwood's machine," J. Phys. (France) **46** (12), 2053-6.

Tufillaro, N. (1985), "Motions of a Swinging Atwood's Machine," J. Phys. (France) **46** (9), 1495-500.

---

### **-pumping a swing**

[To top?](#)

Burns, J. A. (1970), "More on pumping a swing," Am. J. Phys. **38**, 920-922.

Case, W. B. (1996), "The pumping of a swing from the standing position," Am. J. Phys. **64** (3), 215-20.

Case, W. B. and M. A. Swanson (1990), "The pumping of a swing from the seated position," Am. J. Phys. **58**, 463-467.

Curry, S. M. (1976), "How children swing," Am. J. Phys. **44**, 924-926.

Gore, B. F. (1970), "The child's swing," Am. J. Phys. **38**, 378-379.

Gore, B. F. (1971), "Starting a swing from rest," Am. J. Phys. **39**, 347.

Siegmán, A. E. (1969), "Comments on pumping a swing," Am. J. Phys. **37**, 843-844.

Tea, P. L., Jr. and H. Falk (1968), "Pumping on a swing," Am. J. Phys. **36**, 1165-1166.

---

### **-parametric instability**

[To top?](#)

Adler, L. and M. A. Breazeale (1971), "Parametric phenomena in physics," Am. J. Phys. **39**, 1522-1527.

Berthet, R., A. Petrosyan, and B. Roman (2002), "An analog experiment of the parametric instability," Am. J. Phys. **70** (7), 744-749.

Butikov, E. I. (2001), "On the dynamic stabilization of an inverted pendulum," Am. J. Phys. **69** (7), 755-768.

Case, W. (1980), "Parametric instability: an elementary demonstration and discussion," Am. J. Phys. **48**, 218-221.

Cayton, T. E. (1977), "The laboratory spring-mass oscillator: an example of parametric instability," Am. J. Phys. **45**, 723-732.

Curzon, F. L., A. L. H. Loke, M. E. Lefrancois, and K. E. Novik (1995), "Parametric instability of a pendulum," Am. J. Phys. **63** (2), 132-6.

Denardo, B., J. Earwood, and V. Sazonova (1999), "Parametric instability of two coupled nonlinear oscillators," Am. J. Phys. **67** (3), 187-95.

Falk, L. (1979), "Student experiments on parametric resonance," Am. J. Phys. **47**, 325-328.

Fameli, N., F. L. Curzon, and S. Mikoshiba (1999), "Floquet's theorem and matrices for parametric oscillators: Theory and demonstrations," Am. J. Phys. **67** (2), 127-32.

- Grosu, I. and D. Ursu (1982), "Simple apparatus for obtaining parametric resonance," Am. J. Phys. **50**, 561.
- Lai, H. M. (1984), "On the recurrence phenomenon of a resonant spring pendulum," Am. J. Phys. **52**, 219-223.
- Rowland, D. R. (2004), "Parametric resonance and nonlinear string vibrations," Am. J. Phys. **72** (6), 758-66.
- Ruby, L. (1996), "Applications of the Mathieu equation," Am. J. Phys. **64** (1), 39-44.
- Sanmartin, J. R. (1984), "O Botafumeiro: parametric pumping in the Middle Ages," Am. J. Phys. **52**, 937-945.
- Stockman, H. E. (1965), "The electric bell as amplifier," Am. J. Phys. **33**, 505.
- Yorke, E. D. (1978), "Square-wave model for a pendulum with an oscillating suspension," Am. J. Phys. **46**, 285-288.
- Bae, S. (2006), "Equivalence of the pumping of a swing and the parametric resonance," Eur. J. Phys. **27** (2), 291-8.
- Bae, S. and Yoon-Hwan Kang (2006), "Optimal pumping in a model of a swing," Eur. J. Phys. **27** (1), 75-86.
- Butikov, E. I. (2004), "Parametric excitation of a linear oscillator," Eur. J. Phys. **25** (4), 535-54.
- Butikov, E. I. (2005), "Parametric resonance in a linear oscillator at square-wave modulation," Eur. J. Phys. **26** (1), 157-74.
- Leroy, V., J. -C. Bacri, T. Hocquet, and M. Devaud (2006), "A Hamiltonian approach to the parametric excitation," Eur. J. Phys. **27** (3), 469-83.
- Tufillaro, N. B. (1990), "Torsional parametric oscillations in wires," Eur. J. Phys. **11**, 122-124.

---

### **-instabilities, bifurcations & catastrophes**

[To top?](#)

- Case, W. B. (1994), "Time-delay oscillator and instability: a demonstration," Am. J. Phys. **62** (3), 227-30.



- Duffy, B. R. (1993), "A bifurcation problem in hydrostatics," *Am. J. Phys.* **61**, 264-269.
- Duffy, B. R. (1993), "A bifurcation problem in hydrostatics," *Am. J. Phys.* **61** (3), 2.
- Johnson, R. C. (1998), "Unicycles and bifurcations," *Am. J. Phys.* **66** (7), 589-92.
- Litherland, T. J. and A. Siahmakoun (1995), "Chaotic behavior of the Zeeman Catastrophe Machine," *Am. J. Phys.* **63** (5), 426-31.
- Mancuso, R. V. (2000), "A working mechanical model for first- and second-order phase transitions and the cusp catastrophe," *Am. J. Phys.* **68** (3), 271-7.
- Mancuso, R. V. and G. A. Schreiber (2005), "An improved apparatus for demonstrating first- and second-order phase transitions: ball bearings on a rotating hoop," *Am. J. Phys.* **73** (4), 366-7.
- Moisy, F. (2003), "Supercritical bifurcation of a spinning hoop," *Am. J. Phys.* **71** (10), 999-1004.
- Rodewald, B. and H. J. Schlichting (1985), "A catastrophic toy," *Am. J. Phys.* **53**, 1172-1174.
- Brito, L., M. Fiolhais, and J. Paixao (2003), "Cylinder on an incline as a fold catastrophe system," *Eur. J. Phys.* **24** (2), 115-23.
- Chialvo, D. R., A. Vinet, D. Michaels, and J. Jalife (1991), "Bifurcations in a simple hydraulic oscillator: the 'Tantalus cup'," *Eur. J. Phys.* **12**, 297-302.
- Denny, M. (2002), "Watt steam governor stability," *Eur. J. Phys.* **23** (3), 339-51.
- Pippard, A. B. (1980), "Demonstration experiments in critical behavior and broken symmetry," *Eur. J. Phys.* **1**, 13-18.
- Pippard, A. B. (1990), "The elastic arch and its modes of instability," *Eur. J. Phys.* **11**, 359-365.
- Sivardiere, J. (1997), "Simple mechanical systems exhibiting instabilities," *Eur. J. Phys.* **18** (5), 384-7.
- Livesley, D. M., C. J. Brixton, and A. M. Dingley (1984), "The rocking bucket - a simple example of critical behavior," *Phys. Educ.* **19**, 297-301.

## **-chemical and biological oscillators & reaction/diffusions systems**

[To top?](#)

Mielczarek, E. V., J. S. Turner, D. Leiter, and L. Davis (1983), "Chemical clocks: experimental and theoretical models of nonlinear behavior," *Am. J. Phys.* **51**, 32-42.

Yoshikawa, K., N. Oyama, M. Shoji, and S. Nakata (1991), "Use of a saline oscillator as a simple nonlinear dynamical system: rhythms, bifurcation, and entrainment," *Am. J. Phys.* **59**, 137-140.

Fernandez-Garcia, G., M. Gomez-Gesteira, A. P. Munuzuri, V. Perez-Munuzuri, and V. Perez-Villar (1994), "A method for spiral wave generation in the Belousov-Zhabotinsky reaction," *Eur. J. Phys.* **15** (5), 221-7.

Sobrinho, T., M. Alonso, V. Resez-Munusuri, and V. Perez-Villar (1993), "Reaction-diffusion process in a one-dimensional array of active non-linear circuits," *Eur. J. Phys.* **14**, 74-79.

## **-other pattern forming systems & self-organized criticality**

[To top?](#)

Grumbacher, S. K., K. M. McEwen, D. A. Halverson, D. T. Jacobs, and J. Lindner (1993), "Self-organized criticality: an experiment with sandpiles," *Am. J. Phys.* **61**, 329-335.

Grumbacher, S. K., K. M. McEwen, D. A. Halverson, D. T. Jacobs, and J. Lindner (1993), "Self-organized criticality: an experiment with sandpiles," *Am. J. Phys.* **61** (4), 329-35.

Murai, N. and T. Nakata (1988), "Rounded spikes of kompeitoh and scaling relations," *Am. J. Phys.* **56**, 459-462.

O'Keefe, R. (1994), "Modeling the tearing of paper," *Am. J. Phys.* **62** (4), 299-305.

Pritchett, T. and J. K. Kim (1998), "A low-cost apparatus for the production of surface wave patterns in a vertically oscillating fluid," *Am. J. Phys.* **66** (9), 830-3.

## **-miscellaneous nonlinear & chaotic systems**

[To top?](#)

Arnold, T. W. and W. Case (1982), "Nonlinear effects in a simple mechanical system," *Am. J. Phys.* **50**, 220-224.

Ballico, M. J., M. L. Sawley, and F. Skiff (1990), "The bipolar motor: a simple demonstration of deterministic chaos," *Am. J. Phys.* **58**, 58-61.

Cervellati, R. and R. Solda (2001), "An alternating voltage battery with two salt-water oscillators," *Am. J. Phys.* **69** (5), 543-5.

Kautz, R. L. and B. M. Huggard (1994), "Chaos at the amusement part: Dynamics of the Tilt-A-Whirl," *Am. J. Phys.* **62** (1), 59-66.

Meissner, H. and G. Schmidt (1986), "A simple experiment for studying the transition from order to chaos," *Am. J. Phys.* **54**, 800-4.

Mendelson, K. S. and F. G. Kariotis (1991), "Chaoticlike motion of a linear dynamical system," *Am. J. Phys.* **59**, 221-224.

Ojha, A., S. Moon, B. Hoeling, and Siegel P. B. (1991), "Measurements of the transient motion of a simple nonlinear system," *Am. J. Phys.* **59**, 614-618.

Romer, R. H. (1993), "Reading the equations of physics and confronting the phenomena - the delights and dilemmas of physics teaching," *Am. J. Phys.* **61**, 128.

Warden, J. A. (1970), "Demonstration of amplitude jumps," *Am. J. Phys.* **38**, 773-774.

Yoshikawa, K., N. Oyama, M. Shoji, and S. Nakata (1991), "Use of a saline oscillator as a simple nonlinear dynamical system: rhythms, bifurcation, and entrainment," *Am. J. Phys.* **59**, 137-140.

Denny, M. (2004), "Stick-slip motion: an important example of self-excited oscillation," *Eur. J. Phys.* **25** (2), 311-22.

Denny, M. (2005), "The dynamics of antilock brake systems," *Eur. J. Phys.* **26** (6), 1007-16.

Eckert, M. (1996), "The Sommerfeld effect: theory and history of a remarkable resonance phenomenon," *Eur. J. Phys.* **17** (5), 285-9.

Silverman, M. P., W. Strange, and T. C. Lipscombe (1998), "'String theory': equilibrium configurations of a helicoiseir," *Eur. J. Phys.* **19** (4), 379-87.

Viet, O., Wesfreid, and E. Guyon (1983), "Art cinetique et chaos mecanique," *Eur. J. Phys.* **4**, 72-76.

Chacon, R., Y. Batres, and F. Cuadros (1992), "Teaching deterministic chaos through music," Phys. Educ. **27**, 151-154.

[To top?](#)